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INTEGRATING CHRISTIAN VALUES AND LEARNING IN THE TEACHING OF MATHEMATICS

by

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INTRODUCTION

The search for the ultimate meaning of life, which is in fact a problem of value, has always confronted man. But at no other time in history has there been such an unequaled importance given to the question of meaning or to problems of value. It is an accepted fact that clearly defined values give meaning to human activity. A system of values permits man to coordinate his experiences into a meaningful unity encompassing his whole life. (1)

The various fields of human thought and behaviors are all involved in this search for values that give meaning to the human adventure in the changing conditions of society in which man finds himself. Among these various fields of human endeavor, education stands out as one of those fertile grounds for the realization of values in the individual as well as in society. Education is in fact viewed as an exciting project of values in need of realization. Ravitch maintains that "all education implies transmission of values." (2) It is important to note, however, that not all values are moral values. Skulason (1988), in his unpublished manuscript "Minnispunktar um gildismat," distinguished three types of values: moral values that deal with issues like quality of life, freedom, and dignity; worldly values that are about money, power, and fame; and spiritual values that cover areas like understanding and art.

John Dewey explicitly stated: "What we need in education is a genuine faith in the existence of moral principles which are capable of effective application." (3) Moral and spiritual values need to be stressed in our educational institutions. In the present society, no teacher can afford to remain complacent and unconcerned. The times call for action but action, isolated and apart from the others, becomes meaningless and futile. If we have to create an impact in the strengthening of moral values, the millions of teachers, which include the mathematics teachers, must have some form of coordination. This coordinated effort becomes the basis of success.

Purpose of the Study

Mathematics is a part of the elementary, secondary, and college curricula. Every student is required to take mathematics subjects from the grades to the tertiary level. The influence of a mathematics teacher on the thinking of his students cannot be denied. A concerned mathematics teacher, therefore, will take full advantage of these opportunities in making an indelible imprint on the value system of the students under his care.

This paper focuses on how values integration can be done in a mathematics classroom. Specifically, it aims to answer the following questions:

1. What are the moral, social, and spiritual values that can be integrated in mathematics teaching from an Adventist perspective?

2. How can mathematics be made interesting and relevant?

VALUES INTEGRATION IN MATHEMATICS

Mathematics is a necessary human activity, one that reflects a response to needs dictated by human existence itself. Mathematics is evident in all societies and cultures. The needs to which it responds are both material and intellectual, and as they change, so does the nature of mathematics that serves them; thus mathematics is a body of knowledge that is constantly evolving in response to societal conditions. (4)

However, the teaching of mathematics is made one-dimensional. Mathematics teachers frequently find themselves concentrating on the teaching of mathematics - the symbols, the mechanics, the answer-resulting procedures -- without really teaching what mathematics is all about - where it comes from, how it was labored on, how ideals were perceived, refined and developed into useful theories - in brief, its social and human relevance. At best, this practice will produce knowledgeable technicians who can dispassionately use mathematics, but it will also produce students who perceive mathematics as an incomprehensible collection of rules and formulas that appear in mass and threateningly descend on them. These students build psychological barriers to true mathematical understanding and develop anxieties about the learning and use of mathematics. Mathematics has become a dreaded subject in many educational institutions in the world today.

"Why do you teach mathematics?" "Why do we learn mathematics?" These questions bother many mathematics teachers. The answers to these questions provide opportunities for teachers to integrate values in the teaching of mathematics.

1. **Mathematics is useful**. As a rule, students readily become interested in things which are new or exciting, in things which they can perceive practical values or applications to situations and fields of study in which they are already interested, and in things which involve puzzle or elements of mystery. It may be taken as axiomatic that students will work most diligently and most effectively at tasks in which they are genuinely interested. (5)

A good mathematics teacher must be able to point out the relation of mathematics to other fields of study and its applications to fields of work through which people gain their livelihood. Students look forward to the intensely practical problem of selecting an occupation and earning a living and they are generally interested in learning something about the opportunities and requirements in different fields.

It is the responsibility of a mathematics teacher to emphasize continually that mathematics is an essential part of culture and education to understand the background and the nature of the developments which are going on in the world. Students should be impressed with the fact that many of these important developments which directly affect our daily lives cannot be adequately understood except through an understanding of scientific principles whose development, expression and interpretation depend in turn upon mathematical principles. They should be led to see that mathematics will aid them even in such matters as interpreting social and economic phenomena and that it is indispensable to the understanding and development of scientific theory. (6)

"In the study of figures the work should be made practical." (7) The mathematics teacher should be creative enough to formulate real-life situation problems. Environmental problems can be quantified by statistics and graphs. The value of peace can be emphasized through the statistical analysis of the arms expenditure of the world and the awesome, destructive power of nuclear weapons. Let every student be taught not merely to solve imaginary problems but to keep an accurate account of his own income and expenses. Let him learn the right use of money whether supplied by his parents or by his own earnings. Rightly directed, the study of mathematics will encourage habits of benevolence, thrift, self-reliance, conservation of resources, and economic efficiency.

One of the approaches which has been taken to the integration of Christianity and mathematics, though inadequate, is the applicational approach. It argues that "mathematics is useful for Christians in daily life. . . The other side of the same coin is that Christianity is useful in mathematics: honesty in reporting statistics, humility in teaching mathematics, or thankfulness for the beauty of mathematics." (8)

2. **Mathematics disciplines the mind.** "The main design of studying mathematics should be to call into exercise, to discipline, to invigorate the powers of the mind. It is the logic of the mathematics which constitutes their principal value as a part of a course of collegiate instruction. The time and attention devoted to them is for the purpose of forming sound reasoners, rather than expert mathematicians." (9)

Mathematics is done because of the great amount of practice it affords the mind; it is a sort of mental jogging to build up the mind and keep it fit. Accordingly, mathematics textbooks -- the barbells and skipping ropes for the mind -- were designed to provide this needed exercise.

There is disciplinary value in the study of mathematics -- in the development of sound work habits, the capacity to work independently, and the acquiring of problem-solving skills and strategies. There is indeed a wealth of self-discipline which attends the analysis of a problem, the identification of what is given and what is to be solved, the selection of a strategy to solve the problem, and the interpretation of the obtained results. The resulting sense of accomplishment can be enormously satisfying to student and teacher alike.

In teaching mathematics, "let the child and the youth be taught that every mistake, every fault, every difficulty, conquered, becomes a stepping-stone to better and higher things. It is through such experiences that all who have ever made life worth living have achieved success." (10) In learning mathematics, patience and accuracy are developed.

Mathematics is a subject built on the discovery of creative thinkers. In many ways, it offers unique opportunities for creative and original thinking. Writing and solving original problems, establishing theorems with original proofs, discovering and stating relationships in one's own words are beginning experiences in creative thinking. A further opportunity for originality is found in the communication of mathematical ideas, be it in a demonstration, a proof, an exhibit, a poem, or a research project.

A mathematics teacher who desires to produce creative thinkers should consider the following principles:

> If education is seen as the transmission of old knowledge into new heads, then creative thinking has no place. But if education is the systematic encouragement of the human potential to find new ways of living -- of surviving -- then creative thinking is central to the function of the school. Making the classroom an attractive and comfortable place for human beings is to send messages to the people in the room -- messages of safety, security, belongingness, and warmth, messages which say that this is a place where the individual is respected and trusted, where human beings may engage in human activity. This kind of classroom, a humane classroom, fosters an atmosphere in which true creative thinking can take place. (11)

Mathematics is especially appropriate for teaching critical thinking because of the precise nature of the content and the possibility of building logical arguments for and against certain points of view using measurable data.

> Every human being is endowed with a power to think and to do. It is the work of true education [reflected in mathematics teaching] to develop this power, to train the youth to be critical thinkers, and not mere reflectors of other men's thought...Institutions of learning may send forth men

> strong to think and to act, men who are masters and not slaves of circumstances, men who possess breadth of mind, clearness of thought, and the courage of their convictions. Such an education provides more than mental discipline. It strengthens the character so that truth and uprightness are not sacrificed to selfish desire or worldly ambition. (12)

"Happy is the teacher who has learned to lead his or her students in the development of their God-given abilities to think critically and to make conscious ethical choices." In the critical thinking process, however, "the committed Christian teacher will openly acknowledge that there are limits to even the best-trained human reason. He will joyfully exercise to the maximum his God-given rationality while humbly recognizing his need for the guidance of the Holy Spirit in his search for 'all truth'." (13)

3. **Mathematics is beautiful.** The beauty of mathematics has long been an important justification for studying the subject. Hardy asserts: "The mathematician's patterns, like the painters' or the poets', must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics." (14) It is essential that in teaching mathematics, the teacher makes every effort to ensure that the power of the ideas he is dealing with does not escape his students. Mathematics is filled with ideas that are fascinating.

Mathematics is an art and as in any other art, beauty in mathematics consists in order and inner harmony. The mathematician tries to express a maximum number of ideas and relations with the greatest economy of means. The beauty of mathematics can be found in the process whereby a chaos of isolated facts is transformed into a logical order. The exploration of new ideas, the invention of new mathematical structures, is a challenge to the creativeness, the imagination, and the intuition of the mathematician. (15)

Mathematics is the study of patterns--that is, of any kind of regularity in form or idea. Radio waves, molecular structures, and orbits of celestial bodies--all have patterns that can be classified mathematically.

The pattern of adding successive terms 1, 1, 2, 3, 5, 8, 13, 21, 34,. . . suggests a famous sequence called the Fibonacci sequence. The numbers of this sequence are called Fibonacci numbers, and they have interested mathematicians for centuries. In the thirteenth century, Leonardo Fibonacci wrote a book discussing the advantages of Hindu-Arabic numerals over Roman numerals where he had a problem that related Fibonacci numbers to the birth patterns of rabbits: "Suppose a pair of rabbits will produce a new pair of rabbits in their second month and thereafter will produce a new pair every month. The new rabbits will do exactly the same. Starting with one pair, how many pairs will there be in 10 months if no rabbits die?" (16) To solve the problem, Leonardo looked for a pattern, thus, Fibonacci sequence was discovered.

Suppose we consider the ratios of the successive terms of the Fibonacci sequence: 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21,. . . If we continue finding these ratios, you will notice that the sequence oscillates about a number approximately equal to 1.618 known as the golden ratio.

Many studies of the human body itself involve the golden ratio. The ratio of the distance from the navel to the floor to the distance from the top of the head to the navel, the ratio of the height of the person to the distance from the navel to the floor, and the ratio of the arm length to the shoulder width approximate the golden ratio. (17)

Many everyday rectangular objects have a length-to-width ratio of about 1.6:1. The dimensions of the Parthenon at Athens have a length-to-width ratio that almost exactly equals the golden ratio. Psychologists have tested individuals to determine the rectangles they find most pleasing. The results are those rectangles whose length-to-width ratios are near the golden ratio. Such rectangles are called golden rectangles.

The golden ratio is important to artists in a technique known as "dynamic symmetry". Albrecht Durer, Leonardo da Vinci, Georges Seuurat, George Bellows, and Pieter Mondriaan all studied the golden rectangle as a means of creating dynamic symmetry in their work. In fact, Mondriaan is said to have approached every canvass in terms of the golden ratio.

4. <u>Mathematics gives insights into God's creation</u>. One aspect of the beauty of mathematics is found in the world around us. "Everywhere in nature there are evidences of mathematical relationships." (18)

> The Greeks identified mathematics with the reality of the physical world and saw in mathematics the ultimate truth about the structure and design of the universe. They founded the alliance between mathematics and the disinterested study of nature, which has become the very basis of modern science. Moreover, they went far enough in rationalizing nature to establish the conviction that the universe is indeed mathematically designed, that it is controlled, lawful, and intelligible to man. (19)

The hexagons of snowflakes, the crystal structures of minerals, and the hexagonal prisms of the honeycomb are all excellent subjects for geometric study. "All are familiar with the endless variety and perfect symmetry that God has put into the tiny snow-flake. The endlessly embellished hexagons, never repeating themselves, show the infinite variety and creativity of our Master Designer." (20)

"Much mathematics can be gleaned from a study of the cells of a honeycomb. Each cell is a hexagonal prism with a trihedral base formed by three congruent rhombi. It can be shown using methods of the calculus that for a given volume the minimum surface of the prism is achieved when the acute angles of the rhombus are 70 32', which is the angle the bees use in constructing their cells." (21)

Entire books have been written about the bee and how God has given this small insect the ability to build a honeycomb using the smallest amount of material to enclose the greatest possible amount of honey. (22) The bees' accomplishment is as rationally comprehensible as that of a structural engineer. Thus, where the Psalmist saw God's handiwork in the heavens, His wisdom can also be seen among His humbler creatures.

With perhaps the exception of the circle and the ellipse, few curves have been given such a philosophical and mystical significance as the logarithmic spiral. Also known as the equiangular spiral, it is not only a curve of great aesthetic beauty, but its properties make it unique among plane curves. It appears in several instances in nature, sometimes with remarkable accuracy, such as in the nautilus shell and sometimes less accurately, as in the sunflower, spiral galaxies, elephants' tusks, horns of wild sheep, canaries' claws, and spines of a pine cone going in opposite directions. "Spirals seem to be a favourite of God." (23)

The occurrence of patterns in nature has held the fascination of mathematicians. An example of Fibonacci numbers in nature is illustrated by a sunflower. The seeds are arranged in spiral curves. If we count the number of clockwise spirals (13 and 21), they are successive terms in the Fibonacci sequence. This is true of all sunflowers, and indeed, of the seed head of any composite flower such as the daisy with a ratio of 21:34.

Figures with rotational symmetry of order 5 are often the basis of nature's pattern. There are many varieties of plants where flowers form regular pentagrams or five-pointed stars. Some starfish are also formed in the shape of the pentagram. The Pythagoreans studied the pentagram and noted that the ratio of several of its dimensions are the golden ratio. (24)

The common geometric forms such as squares, triangles, rectangles, and hexagons, appear to be the building blocks that God used in creating many things. An example of this is found in viewing a cross-section of a semiprecious gem called tourmaline which is mined in Madagascar. The stone's prismatic structure consists of a series of similar triangles neatly nested one inside the other. God seems to delight in the creation of varied geometric shapes. Common mineral quartz for example often occurs as a triagonal trapezohedron or more simply a crystal structure made up in a triangular arrangement with individual faces being four-sided. (25)

Indeed, patterns in nature can be fully understood through mathematical principles. Rightly taught, numerical and geometric patterns in mathematics will inspire students to appreciate its beauty as well as the beauty of nature.

"From the intrinsic evidence of His creation, the Great Architect of the Universe now begins to appear as a great mathematician." (26) Dirac, a great physicist and mathematician, frankly acknowledges the impossibility of mechanistic explanations for the orderly beauty of the universe:

> It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of a mathematical theory of great beauty and power, needing quite a high standard of mathematics for one to understand it. You may wonder: Why is nature constructed along these lines? One can only answer that our present knowledge seems to show that nature is so constructed. We simply have to accept it. One could perhaps describe the situation by saying that God is a mathematician of a very high order, and He used very advanced mathematics in constructing the universe. Our feeble attempts at mathematics enable us to understand a bit of the universe, and as we proceed to develop higher and higher mathematics, we can hope to understand the universe better. (27)

5. **Mathematics allows us to probe into the mind of God.** "Education is the harmonious development of the physical, mental, and spiritual powers." (28) Mathematics poses a real challenge to the teacher's creativity to establish spiritual links. Each teacher should integrate spiritual associations as the subject matter naturally presents the opportunity. This integration must not be artificial, forced, gimmicky, or tacked on. Through the teacher's ingenuity, the students will be led to develop creativity and spiritual awareness.

The study of mathematics seems to point to the changelessness of God. It reveals the wisdom of God, and that He is a God of order and system. When the students learn mathematical processes, axioms and laws, they may be further enabled to more clearly identify God's design and handiwork in nature. Byrne wrote: "Mathematics is a revelation of the thought life of God. It shows Him to be a God of system, order, and accuracy. He can be depended upon. His logic is certain. By thinking in mathematical terms, therefore, we are actually thinking God's thoughts after Him." (29)

Many analogies can be made between mathematics and life. Consider the following religious concepts that can be integrated in the teaching of mathematics:

1. God is the Designer, Creator and Sustainer of order, precision, perfection and beauty.
2. Things of nature give evidence that God enjoys symmetry and geometric design.
3. The logical development of a mathematical system illustrates the thought process used in developing a systematic theology.
   a. In logic, the premises accepted determine the conclusions reached. In Bible doctrines, false premises lead to wrong conclusions even when they are reached by correct logic.
   b. Conditionals used in proofs remind us that many of God's promises are also conditional.
4. Mathematics illustrates the importance of little things.
   a. In mathematical symbolism, even the slightest error can make a great difference in the answer. Similarly in life, small faults uncorrected make vast difference in one's character development.
   b. However small the measure of an angle, the farther from the vertex, the farther the legs diverge. Also, even the slightest divergence from the correct path in life eventually widens with time.
5. Mathematical concepts illustrate God's nature.
   a. God's faithfulness and dependability are illustrated in the constancy and predictability of mathematical rules.
   b. The real number line has no beginning nor ending -- similar to the infinity of God.
   c. Many of God's laws of nature can be expressed in mathematical models which are always self-enforcing. (30)

There are obvious dangers and a great deal of reluctance associated with overtly deciding to what values, rank order and mix of values and associated value-related beliefs and attitudes we want our students to commit their allegiance to. We have been committing them, largely unthinkingly and unknowingly, to our own values and the values of the culture at large. This is an even more dangerous practice; the values of the culture or even our own values may not be the values we would want our students to give allegiance to, or we may be sending them mixed signals; by our own words we may be suggesting one thing and by our actions we may be appearing to condone something else. A better option would be to bring values out into the open and to educate students so that they can exercise critical, reflective judgment about the value to which they wish to subscribe and that are best for themselves and the rest of humanity. Through the guidance of the Holy Spirit, a Christian mathematics teacher can greatly influence the value system of his/her students.

MAKING MATHEMATICS TEACHING RELEVANT AND INTERESTING

Many people seem to believe that mathematics is a mysterious force to be understood by a chosen, gifted few. Teachers are aware that there is a established fear of mathematics among students which results in underachievement. This is a pressing problem in mathematics education around the world.

Teachers can remedy this situation by incorporating a historical perspective into the teaching of mathematics to lessen its stultifying mystique. Mathematics teaching can be humanized by the inclusion of historical perspectives in classroom discussion.

1. History of Mathematics

Historically, the study of mathematics arose from observations of nature. Early mathematicians believed that nature imposed the mathematics upon mankind. A piece of mathematics was valid to the extent that it agreed with nature. Galileo Galilei (1564-1643) wrote: "The universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language." (31) Thus the study of mathematics was considered the study of God and creation. "To learn mathematics is to discover the very mind of God." (32)

Mathematics, like astronomy, arose in Babylon before it developed in Greece. In Babylon, as later in Greece, it was the religious interest in the heavens that led to the study of the stars and to the quantification of their movements and distances by means of mathematics. Jacob Bronowski proposes that the reason astronomy as a science developed ahead of other sciences was that the observed motions of the stars turned out to be calculable and lent themselves to mathematics. (33) The Bible asserts that "the heavens declare the glory of God; the skies proclaim the work of His hands." (Ps. 19:1), so it is natural that science and mathematics had many of their roots in astronomy.

The well-known quote from Plato that "God ever geometrizes" reflects the reverence with which the ancients approached the science of mathematics. For the Pythagoreans, numbers were not only integers for counting and a means for calculating astronomical ratios, but were of the essence of reality and the focus of their faith. Numbers were not simply symbols for measuring reality; they were reality. (34)

For the Pythagoreans, mathematics embraced the heavens and the earth, divinity, and creation. For them as for Leopold Kronecker (1823-91), pioneer in the field of algebraic numbers, "God is responsible for making the numbers. Everything else is of human creation." (35)

Archytas (c.430-c.360 B.C.) reflected fascination verging on fanaticism when he says that only mathematicians, those "who have gained an exact knowledge of the universe and are concerned with the two related primitive forms of being (numbers and form), are capable of true apprehension of reality. They alone have handed down to us a clear insight into the speed of the stars, about their rising and setting, and about geometry, numbers (arithmetic), sphericity and, not least of all, about music as well." (36)

During the late Middle Ages, science and mathematics were also applied extensively to theological problems that were largely or wholly unrelated to the creation account in Genesis. Themes, techniques and ideas from natural philosophy and mathematics were frequently used in problems that concerned God's omnipresence, omnipotence, and infinity, as well as His relation to the beings of His own creation and to comparisons between created species. Mathematical concepts were regularly drawn from proportionality theory, the nature of the mathematical continuum, convergent and divergent infinite series, the infinitely large and small, potential and actual infinites, and limits, which included boundary conditions involving first and last instants or points. (37)

Also, theological concepts were formulated in the language of mathematics and measurements. Such concepts were employed to describe the manner in which spiritual entities could vary in intensity and how such variations could best be represented mathematically by application of the peculiarly medieval doctrine known as "the intention and remission of forms of qualities." They were also to determine the manner in which upper and lower limits, or first and last instants, could be assigned to various processes and events, as in problems concerning free will, merit, and sin. (38)

The mathematical concepts were applied to many other problems including speculations about God's infinite attributes namely his power, presence, and essence, the kinds of infinites He could possibly create, the possible eternity of the world, the behavior of angels, etc. In contemplating the range of theological topics to which mathematics and mathematical concepts were applied, one may reasonably conclude that in the 14th century, theology had been quantified. (39)

Mathematical theory "would most readily prepare the way to the theological, since it alone could take good aim at that unchangeable and separate act, so close to that act are the properties having to do with translations and arrangements of movements, belonging to those heavenly beings which are sensible and both moving and moved, but eternal and impassible." (40)

Ptolemy was certain that mathematics was universally relevant and that it mediated between the divine and the physical. By mathematics one could know not only the sameness, good, order, and true proportions but the simple directness contemplated in divine things, making its followers lovers of that divine beauty, and making habitual in them and as it were natural, a like condition of the soul. (41)

2. **Biographical Sketches of Mathematicians**

One of the value teaching strategies is to "use the history of mathematics to illustrate biographical values in the lives of mathematicians." (42) Mathematics is not something magic and forbiddingly alien, but rather it is a body of knowledge naturally developed by people over a 5000-year period--people who made mistakes and were often puzzled but who worked out solutions to their problems and left records of these solutions so that we can benefit from them. Its teaching should recognize and promote these people-centered facts.

History of mathematics tells us that many mathematicians were theologians. Many of them believe in the God of heaven. Newton, for example, was an unquestioning believer in an all-wise Creator of the universe and even though he was the greatest mathematician of his time, he was also a believer in his inability to comprehend the universe. He wrote: "I do not know what I may appear to the world but to myself I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me." (43)

The virtues of these mathematicians are worthy of discussion. It should be done in a natural way as an integral part of the lesson. This can, and should be done unobtrusively by the use of historical anecdotes, films, projects, displays, and problems in teaching presentations as well as the replication of relevant historical experiments.

3. **Making Mathematics Fun**

A mathematics teacher needs some serious analysis of what and how he can exploit various facets of the nature of mathematics to achieve desirable educational goals in the affective domain. But his work in the affective domain should be geared to learning more mathematics through liking mathematics more. (44) If he cannot make the subject intrinsically interesting, he should try to add excitement by means of games, story-telling from the history of mathematics and field trips to places where mathematics is being used. Lessons can be dressed up with puzzles, number tricks, paradoxes, or alphametics.

Anota made a study of the effect of affect-oriented mathematics lessons on student's attitudes toward and achievement in mathematics. "Affect-oriented mathematics lesson is a lesson that integrates activities designed to develop the student's interest in and appreciation for mathematics besides the development of mathematics concepts." (45) He found out that the use of affect-oriented mathematics lessons effected better learning in and a more favorable attitude toward mathematics among the students. He therefore recommended that affect-oriented mathematics activities should be used not only as motivational or recreational devices, but also as enrichment activities to help in the development of mathematics concepts and computational skills.

4. **Use of Cooperative Learning Models**

When students are working on mathematics problems, they need to verbalize what they are doing in order to connect appropriate words and symbols one-on-one with the teacher. The solution is for students to work on their assignments in cooperative-learning groups of two to six.

In classrooms using cooperative-learning groups, students do not sit quietly and listen to the teacher's instructions and then do their work independently. Rather, they work in small groups on a common task that draws on each group members' strengths. Teachers must provide instruction and practice in the social skills needed for effective group cooperation and provide time for students to evaluate how well they are using these skills. Further, teachers must develop assignments that require each member of the group to work cooperatively with others. In cooperative learning, the group assumes the responsibility of ensuring that each member understands the math problem solution before turning in the group assignment.

Cooperative learning methods have considerable support from research. In a meta-analysis of 122 studies on cooperative learning, Johnson et. al. conclude: "Cooperative learning experiences tend to promote higher achievement than do competitive and individualistic learning experiences. These results hold for all age levels, for all subject areas, and for tasks involving concept attainment, verbal problem-solving, categorization, spatial problem-solving, retention and memory, motor performance, and guessing-judging-predicting." (46)

Cooperative learning experiences not only result in higher achievement but also develop the students' social skills as acceptance, mutual cooperation, respect for self and others, accepting responsibility, sharing, reasoning process, following instructions, listening, tolerance, and open-mindedness.

5. Reducing the Fear of Mathemathics

Attitudes are fundamental to the dynamics of behavior. They largely determine what students learn. The mathematics student with positive attitudes studies mathematics because he enjoys it, he gets satisfaction from knowing mathematical ideas, and he finds mathematical competency its own reward. On the other hand, no student can be forced to learn mathematics which he doesn't want to learn. This negative attitude leads to failure, failure to dislike, dislike to fear, and fear to hatred of the subject.

The mathematics teacher cannot choose to avoid the problem of developing positive attitudes toward mathematics. "Whether the teacher likes it or not, each student responds emotionally to his teachers." (47) Students react strongly, acquiring or rejecting on the basis of this response the attitudes, values, and appreciation of the teacher. The teacher is the key factor for the development of his students' attitudes toward the subject he teaches.

The teacher's appreciation of mathematics as an important, dynamic, remarkable subject must be real and deep, his attitude toward students must be sympathetic and understanding, his interest in learning must be great, his enthusiasm for teaching sincere. If the teacher's attitudes are less favorable or his motivation the same as that of the student, no transmission of enthusiasm can take place.

Once the teacher has set his own standards high, he must still establish himself as a person who has the respect and esteem of his students. Maintaining the students' esteem in the face of their constant judgmental response is a sobering problem for the teacher. Students are quick to sense the smallest insincerity just as they note the slightest insecurity. It is extremely important that the classroom atmosphere be friendly, accepting, and supportive, even when it is demanding and challenging. A spirit of security, enjoyment, and loyalty should be the basic goal of classroom organization. The teacher should make his students feel that his attitude will still be friendly regardless of the success or failure of the students' efforts. If we want our students to think for themselves, we must allow them to try out their own ideas and answers.

For students to build confidence in and loyalty to mathematics, a mathematics teacher needs to be the kind of person they accept and are willing to imitate, to work with them with patience and kindness so that each day each student has some success, and to make learning mathematics a privilege rather than a punishment. To establish an optimistic attitude, he needs to present problems in a way that does not threaten the student's ego, to assign tasks that are within the range of the student's ability, and to be an optimistic, enthusiastic, and sincere person.

Some of the really important ways in which teachers influence their students' attitudes have to do with pleasant communicatiion habits: the teacher's voice inflection, the way he looks at students, his responses, or even his failure to respond. The classroom itself should be attractive. It should contain books, pamphlets, pictures, and displays reflecting an intellectual atmosphere and providing a proper setting for an enriched program.

THE LIMITATIONS OF MATHEMATICS

Mathematics holds a rather unique place in the world today. It is responsible for much of the advancement in science and technology in recent years. Most people accept the fact that it is vital to the continued growth of a nation, both for expanding internal advancement and for the maintenance of a leading role in the world community.

Technology has taken over as the central concern of society. At the same time, people measure social progress in the most quantitative and physical terms. Measures of national welfare are such things as life expectancy at birth and per capita gross national product. What we call standard of living is in reality standard of consumption. Even socialist China, with its newfound focus on materialism, rates its progress in terms of consumer goods: 3.5 wristwatches per family; 1.8 bicycles; 0.2 television sets. These scientifically measurable quantities, simply because they can be measured, are more and more the yardsticks by which a society evaluates itself. (48)

There is danger in quantifying everything. Humans are irreducible. Most of the ethical issues are not quantifiable. Those things that make life worth living--binding friendships, sacred rituals, challenging projects, and the awareness of transcendent beauty--are independent of technology and defy quantification.

It should be noted that mathematics is a creation of the human mind which is itself limited. The wrong use of mathematics may lead people to forget that there is a God who guided mathematicians to discover mathematical knowledge for the benefit of mankind.

CONCLUSION

Mathematics is regarded as a body of experimentally verified knowledge. People think that mathematics has nothing to do with human values, that it is value-free and morally and spiritually neutral. Indeed, the obvious interfaces between mathematics and values are few. The integration of values and learning in mathematics is a challenge to mathematics teachers. Careful study must be given to identify opportunities for integrating values into the mathematics curriculum so that it will be done effectively without the connections appearing artificial and forced. The exploration, assessment, and development of values as an objective of mathematics education is a goal whose time has come.

A Christian mathematics teacher needs to reflect on his delicate responsibility as God's instrument of learning. He who desires the best for his students should live by the following principles:

> The true teacher is not satisfied with second-rate work. He is not satisfied with directing his students to a standard lower than the highest which it is possible for them to attain. He cannot be content with imparting them only technical knowledge. . . It is his ambition to inspire them with principles of truth, obedience, honor, integrity, and purity -- principles that will make them a positive force for the stability and uplifting of society. He desires them above all else, to learn life's great lessons of unselfish service.
>
> True education does not ignore the value of scientific knowledge or literary acquirements; but above information, it values power; above power, goodness; above intellectual acquirements, character." (49)

Mathematics education, being a part of the total educational process, has the same subject, object, and task. The giving of time and space for values education in the mathematics curriculum is a significant breakthrough in curricular reform. Mathematics teachers can help develop in the learners the ability to reason which will assist them to make and judge moral decisions and moral actions.

An in-depth study of ways to integrate values and learning in mathematics will lead to the conclusion that this can be done in the most satisfying way by helping young people take responsibility for their own integration of values and learning. The writer hopes that this paper will form the basis for continued study to discover ways of preparing students to be effective citizens of this world and of the world to come. It is hoped that mathematics teachers will be inspired to use mathematics as an instrument in attaining one of the major goals of education: to preserve, develop, and promote desirable cultural, moral and spiritual values in a changing world.

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