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MATHEMATICAL REASONING: IMPLICATIONS FOR THE CONTEMPORARY CHRISTIAN

by

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INTRODUCTION

In presenting this paper, I have assumed that as Christian Educators we have a Christ centred worldview and for that matter, clearly see and appreciate the need for the integration of faith and learning in the classroom and beyond.

The wise saying that not all that glitters is gold is more relevant in our time and more especially in Ghana than ever before. This is because we are living in a world where almost everything has its imitation. Fake drugs, fake electrical gadgets, fake passports, fake currencies etc are being identified and exposed everyday. Even the spiritual world is not spared at all. There are false prophets, false Christ's, false teachers and false counsellors. This situation calls for mental alertness, vigilance, thoroughness, critical thinking and above all, readiness to apply a proven, consistent and reliable yardstick to whatever ideology or situation we are confronted with. The apostle Paul commended the Christians at Berea for their careful study of the scriptures when he said: "These were more noble than those in Thessalonica, in that they received the word with all readiness of mind, and searched the scriptures daily whether those things were so." (Acts 17:11). The same Paul in I Thessalonians 5:21 states, "Put all things to the test and keep that which is good (T. E. V.). This verse indicates that God values careful thinking. Though the process of testing may be long and painful, it is the most rewarding in the long run because it provides the solid basis upon which we can accept or reject whatever information is before us.

It is against this background that Ellen White says: "Instead of educated weaklings, institutions of learning may send forth men strong to think and to act, men who are masters and not slaves of circumstances, men who possess breadth of mind, clearness of thought and the courage of convictions. Such an education provides more than mental discipline. It strengthens the character so that truth and uprightness are not sacrificed to selfish desire or worldly ambition."

According to Day, "The main design of studying mathematics should be to call into exercise, to discipline, to invigorate the powers of the mind. It is the logic of the mathematics, which constitutes their principal value as a part of a collegiate instruction. The time and attention devoted to them is for the purpose of forming sound reasoners, (emphasis mine) rather than expert mathematicians " (Day, 1923). The above quotations reveal the importance of teaching mathematics in our schools and the need to encourage critical thinking as we teach the subject. To produce such calibre of men and women from our institutions of learning, mathematics cannot be left out. "Indeed the development of reasoning ability is one of the major goals of education. As students develop their reasoning power, they progress toward their highest levels of Bloom's taxonomy and encounter the need to synthesise that what they reasoned will always be true " (Fitzgerald, 1996). "Mathematics provides a way of logical thinking that can be used in understanding how to interpret God's written word, as well as His book of nature. Mathematics should be taught in a way that emphasises this type of thinking and its usefulness in understanding more about God " (Akers, 8CC).

The purpose of this paper is to bring to the fore, the characteristic features of mathematical reasoning and more especially deductive reasoning and the lessons that can be drawn from it to guide instruction in the classroom and Christian life in general. It does so by addressing the following questions:

1. What are the various methods of obtaining knowledge?

2. Why is deductive reasoning (an aspect of mathematical reasoning) of special interest to the mathematician?
3. What useful lessons can Christians in general and mathematics educators in particular draw from mathematical reasoning?

Methods of Obtaining Knowledge

Before we narrow down the discussion to mathematical reasoning, it would be appropriate to refresh our minds of the various ways of obtaining knowledge of which reasoning is a part.

(a) By Intuition: This is the act of knowing without the use of rational processes. It is precognitive apprehension or immediate awareness of reality.

(b) By Authority: This is where the one in search of knowledge on a particular issue turns to an authority on that issue or field of study. This is often the case in obtaining historical knowledge.

(c) By Revelation: By revelation, we mean Divine communication. God the Creator actively disclosing to men his power and glory, his nature and character, his will, ways and plans - in short himself.

(d) By Experience: The road one takes when going to the city centre could be based on experience. Because road A is shorter and virtually free from traffic congestion than road B, one would always prefer to go by road A to B. The decision to go by road A to the city centre instead of B is based on experience.

(e) By Experimentation: This is closely related to experience. It amounts to going through a series of purposive, systematic experiences. Though experimentation is fundamentally experience, it is usually accompanied by careful planning which eliminates extraneous factors and the experience is repeated enough times to yield reliable information.

(f) Reasoning: This is defined as the ability to think logically. Rasi (2001) defines reasoning as "the mental capacity for rational thought, understanding, discernment and acceptance of a concept or idea. It seeks for clarity, consistency, coherence and proper evidence". And within the domain of reasoning there are several forms: - analogy, induction and deduction. Deductive logic is basically treated in Algebra and Discrete Mathematics at the Undergraduate level.

Reasoning by Analogy

A boy who is considering a college career may note that his friend went to college and handled it successfully. He argues that since he is very much like his friend in physical and mental qualities, he too should succeed in college work. This method of reasoning is to find a similar situation or circumstance and to argue that what was true for the similar case should be true of the one in question. This is an example of reasoning by analogy. Of course, one must be able to find a similar situation and one must take the chance that the differences do not matter.

In Acts 19:11 – 16, we are told the story of the seven sons of Sceva who saw apostle Paul healing the sick and casting out demons using the name of Jesus. These young men might have reasoned that Paul was just human like them and since they could also call the name of Jesus, what Paul did, they could also do. When they attempted it, they suffered the consequences of reasoning by analogy.

The Parable that Jesus told about the Rich man and Lazarus in Luke 16:19 – 31 is another example of analogy. A Christian who died does not go immediately to heaven and neither do the wicked immediately go to hell fire. Besides, there is no evidence in scripture to suggest that the righteous and the unrighteous can see and communicate with one another after death.

The risk involved when one reasoned by analogy is where he/she ignores the differences in the two situations. This becomes even more dangerous when extended to spiritual matters. The mathematics educator should therefore help his/her students to see these dangers as they arrive at their conclusions by analogy.

Reasoning by induction

This is the process of forming general conjectures that are based on a number of observations of specific instances. The conjectures may be probable, but they are not necessarily true. People use this method of reasoning everyday. Suppose that every male acquaintance expresses an interest in watching TV broadcast of football games. This information might lead us to conclude that men in general enjoy this activity. With inductive reasoning, however, we can never be absolutely certain that we have reached a correct conclusion. Some day we might meet a man who hates watching TV football. Or, experimentation shows that iron, copper, brass, zinc, oil and other substances expand when heated, one consequently concludes that all substances expand when heated. When in actual fact, water contracts when heated from 0° to 4° centigrade.

Euclid's fifth postulate which is also known as the parallel postulate states that: "if two lines m and l (as shown in fig. 1) meet a third line, n, so as to make the sum of the angles 1 and 2 less than 180°, then the lines m and l meet on that side of the line n, on which the angles 1 and 2 lie."

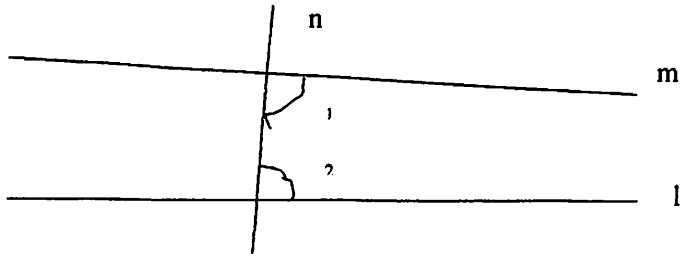


Fig 1 Euclid's Parallel axiom

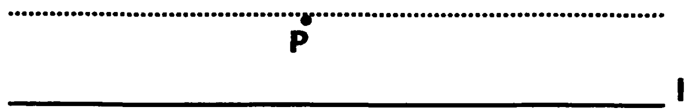


Fig 2 Playfair's parallel axiom

An equivalent to Euclid's fifth axiom is known as Playfair's Parallel axiom which states that, "given a line l (fig 2) and a point P not on that line, there is one and only one line m in the plane of P and l which passes through P and does not meet l."

Euclid's parallel axiom asserts in effect the existence of one and only one line through P and parallel to l. For over 2000 years, this postulate was fully accepted by all mathematicians until 1815 A. D. when János Bolyai and Nicolai Lobachevsky living thousands of miles apart came out with separate studies clearly showing that: (1) There are at least two possible ways that this fifth postulate can be changed. (2) The Euclidean parallel axiom cannot be proved. Morris Kline (1964) quoting Lobachevsky said "Two thousand years of fruitless attempts to put the parallel axiom on an unquestionable basis had led me to suspect that it could not be done". In other words, while Euclid was sure that through a given point, not on a given line, there can only be one line through that point which could be parallel to the given line, it is now known that it is just one alternative.

Relating this experience to our spiritual life, Sire (1990) aptly puts it this way. "When I become so sure that I have it right that I stand against all comers, I come close to standing against God Himself, making him conform to the box in which my concept of him has put him. But God is bigger than my box or anyone's box."

According to Akers et al., (8CC) "A Christian's acceptance of the Bible and the development of his faith is a process that uses some of the same inductive-deductive reasoning and axiomatic systems as mathematics uses. A person can through observation be led inductively to accept the Bible as God's word. This acceptance becomes an axiomatic statement but must be based on faith. As long as that position of faith is maintained, the Bible becomes the a priori basis for careful study, using both inductive and deductive reasoning to better understand God, the great controversy and His plan for man."

Inductive Bible study method as defined in the 1994 edition of the Teacher's Sabbath School emphasises the process of careful and controlled discovery of the meaning of the text. Inductive study could be called "discovery" learning because the student is invited to engage in careful, methodical and intelligent discovery. In inductive approaches, the teacher facilitates and supports the learner's investigation and discovery using seven distinctive approaches:

(a) Inductive Study is methodical: Good Bible readers are like detectives. They sift thoroughly the evidence in a systematic way before drawing a conclusion.

(b) Inductive Study uses careful methods of interpretation: We look for meaning in a careful and thorough manner, understanding the context, taking the words of the passage seriously and avoiding misquoting the author.

(c) Inductive Study is shared study: A wealth of insights comes through group observation as students examine together a Bible passage.

(d) Inductive Study is discussion oriented: Everyone brainstorms, debates, questions, backs and forths, so that all can be involved in the study.

(e) Inductive Study is Scientific: We try to find out what the text says by observing it in context. We do not develop opinions without biblical proof.

(f) Inductive Study is application oriented: We always ask what the text means to us.

(g) Inductive Study focuses on the process and product: Study of and interaction with the text of scripture allows the Spirit of God to minister to us.

Reasoning by Deduction

"According to Logicians, the strongest arguments are deductively valid, which means that it is impossible for the conclusion of an argument to be false if the premises are true" (Skyrms, 1986). At this juncture, what is deductive reasoning?

The process of drawing a conclusion from a sequence of propositions is called deductive reasoning. In deductive reasoning, we start with certain statements, called premises and assert a conclusion, which is a necessary or inescapable consequence of the premises. An example of such an argument is the following:

1. If it's raining, I'll take an umbrella.
2. It's raining.
3. Therefore. I'll take an umbrella.

Statements (1) and (2) are called the premises and (3) the conclusion.

In symbolic language, statements 1,2 and 3 can be written as follows:

p: It is raining.

q: I'll take an umbrella.

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore\ q \end{array}$$

The valid argument illustrated above is known in logic as the Law of Detachment or Modus Ponens.

The Importance of Deductive Reasoning

How important is deductive reasoning? And why is deductive reasoning of special interest to the mathematician? The answers to these questions are not far fetched.

(1) "Simple deductive arguments about right triangles, and admittedly some physical data enabled man to determine the sizes and distances of the heavenly bodies and thus obtain the first real knowledge of the Solar System. Equally valuable results were deduced about the behaviour of light and sound from mathematical and physical principles. Deductive reasoning applied to the laws of electromagnetism and the axiom of number led to the conclusion that electromagnetic waves must propagate through space" (Kline, 1964).

(2) The decision to explore deductively the axioms of a non-Euclidean geometry led to the construction of totally new geometries such as hyperbolic geometry and elliptic geometry which are very useful today.

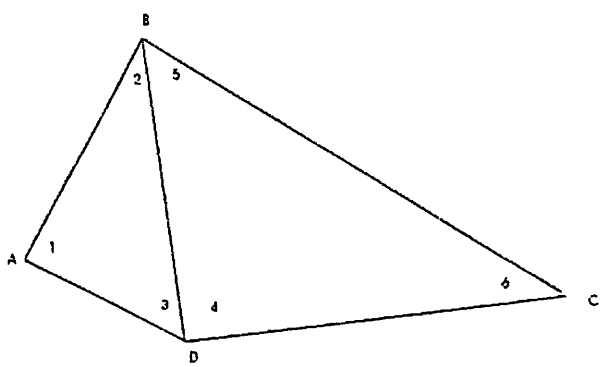
(3) The restriction of mathematics to results deduced from explicit axioms has had several major values. It obliged human beings to apply their God given reasoning faculty. Much of our deepest knowledge, which we would otherwise not have had, was obtained by deductive reasoning.

(4) The fourth value of the deductive process is its power to organise knowledge. If all the results now available in Euclidean geometry had been obtained by a multitude of observations, inductions, or measurements, they would make an unwieldy mass that could not be assimilated. The value of the information would hardly be realised. But deductive organisation permits the mind to survey the whole readily, grasp what is fundamental and what is not, and see the interrelationships of the many conclusions. Comprehension is vastly aided.

(5) Deductive reasoning is important in the study of geometry. As already stated, deductive reasoning begins with statements that are already accepted as true. Those truths are then combined in a logical way to reach a conclusion. When the original statements are true, correct deductive reasoning always leads to true conclusions.

Here is an example of deductive reasoning: Suppose we wish to prove that the sum of the measures of the angles of a quadrilateral (a closed four-sided figure) is $360°$. We could begin with two facts that we know are true: (1) Any quadrilateral can be divided into two triangles and (2) The sum of the measures of the angels of any triangle is $180°$. Reasoning deductively, we may conclude that the sum of the measures of the angles of a quadrilateral must be equal $2 \times 180°$ – that is, twice the sum of the measures of the angles of a triangle, or $360°$. In quadrilateral ABCD below, the measures of angle 1 (symbol $m\angle 1$) $+ m\angle 2 + m\angle 3 = 180°$, and $m\angle 4 + m\angle 5 + m\angle 6 = 180°$.

Thus, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360°$.



Because we have reasoned logically from known facts, we may be certain that our conclusion is true.

Implications

From the above discussions, the following implications for the Christian are clear:

(1). Just as deductive reasoning is cherished by all mathematicians and is used for establishing the truth of arguments because of its dependability and certainty, should remind us of God's faithfulness, dependability, and the certainty of His words. God also expects his workers in any sphere of life to exhibit those same qualities of dependability and certainty. To ensure this, educators in general and in particular, mathematics educators should go beyond leading students to solve mathematical problems. "His students should see him as a caring and loving teacher – caring not just for the subject, but for them as well" (Walthall, 8CC). This should not be an occasional thing. The teacher should be consistently consistent in showing these qualities. This will naturally make his/her students feel welcome all the time to share their joys and problems with him/her.

(2). Correct deductive reasoning always produces the right conclusions when the original statements are true hence the mathematician's love for it. In the same way, Adventist education should produce the right calibre of men and women needed in our time. It cannot afford to fail the world and the Church. Its "reliability coefficient" should be as high as possible. To be able to do this, we need to take a frequent and critical look at what we teach, how we teach it and those who teach it. For the mathematics educator, his/her primary motive should be to equip his/her students to adapt, be creative, think critically and be responsible agents of change wherever they may be found.

(3). Deductive reasoning thrives on premises. A premise is a statement or idea on which reasoning is based. In mathematical reasoning, the premises accepted determine the conclusions reached. In other words, wrong premises would lead to incorrect conclusions. This is what is popularly stated in mathematical circles as follows: "If you accept my premises, then you should also accept my conclusion". In Bible doctrines, false premises lead to wrong conclusions even when the conclusions are reached by correct logic. This is why the premises upon which doctrines are based become very important. When students are led gradually to appreciate this fact they would be careful like the Bereans in their thinking and acceptance of any ideology.

(4). The process of establishing truth by deduction can be long and painful, yet it is a sure way of avoiding errors, since according to Thales, quoted by Morris Kline "the obvious is far more suspect that the obstruse". Let us consider this example about even numbers.

$4 = 2 + 2$	$24 = 5 + 19$
$6 = 3 + 3$	$26 = 13 + 13$
$8 = 3 + 5$	$28 = 11 + 17$
$10 = 3 + 7$	$30 = 13 + 17$
$12 = 5 + 7$	$32 = 13 + 19$
$14 = 7 + 7$	$34 = 17 + 17$
$16 = 3 + 13$	$36 = 17 + 19$

$18 = 5 + 13$

$20 = 3 + 17$

$22 = 11 + 11$

$38 = 19 + 19$

$40 = 17 + 23$

...

Even if one investigates larger and larger even numbers, one finds without exception that every even number can be expressed as the sum of two prime numbers. Hence by inductive reasoning, one could conclude that every even number is the sum of two primes. This conjecture is known as Goldbach's conjecture after the eighteenth century mathematician Christian Goldbach. The fact that this conjecture cannot be proved deductively from acceptable premises makes it still an unsolved problem in mathematics. The mathematician therefore is perceived in some quarters as narrow-minded and shortsighted especially when compared to the scientist who accepts reasoning by analogy as well as by induction to draw formal conclusions.

However, whichever way the mathematician is perceived, he achieves a reputation for certainty. No wonder Morris Kline says "Deductive logic is the hygiene that mathematics practices to keep its ideas healthy and strong." This quotation reminds us also of God's "measuring rod" given to Christians in Isaiah 8: 20 which says, "To the law and to the testimony, if they speak not according to it, it is because there is no light in them." According to God's word, this should be the litmus test for all his children to either accept or reject any doctrine. The individual or the Church may be accused of being narrow-minded or shortsighted but one thing that remains certain is that by holding fast to what God says, his or her conclusions will always stand the test of time.

(5) The danger associated with reasoning by deduction is that: should it be extended "rigidly" to Biblical issues, the mathematician would have to reject all conclusions that cannot be reached deductively. "Students should be helped to understand that the formal language of mathematics is not used in the Bible. The idiomatic expressions and poetic style of the Bible lack the preciseness of mathematical language. Unfortunately, some Christians attempt to use the Bible as though its language were as scientifically precise as that of a well-written mathematics textbook. This has led some to believe that the Bible is inaccurate or internally inconsistent. Galileo clearly recognized this problem when he stated, "For the Bible is not chained in every expression to conditions as strict as those which govern all physical effects: nor is God less excellently revealed in Nature's actions than in the sacred statement of the Bible "" (Akers et al., 8CC).

The New Mind and Mathematical Reasoning

The "new mind" is a term used by scientist of late to refer to young men and women who were born after 1969 (Vogel, 2000). Research has shown that the immense influx of information in today's world coupled with a diminished ability or willingness to evaluate these stimuli, has lead to changes in the way the brain processes and stores information (especially in the new mind). Long term studies have shown how the "new mind" works. Because of parallel circuits and link-ups, it is able to accept and store different stimuli concurrently and independently. The result is an increased acceptance of dissonance. Many "new minds" have grown up with contradictions and are able to handle them – capability of the mind to reconcile the irreconcilable and to give everything equal validity.

According to Kneissler quoted in Vogel, the mind accomplishes this by simply refusing to harmonise or even to evaluate contradictory information. It simply stores contradictory information uncensored. Classical logic has maintained that "a" excludes (not the same as) "non- a". Philosophers and biblical writers like Paul have employed logical reasoning in order to convince others of the correctness of their faith and their belief system. The new logic by the new mind is prepared to question all that. "A" can now be "non-a" as long as the contradiction does not impact adversely on everyday life. The result is pluralism in an individual's thinking.

According to Vogel, the new logic ties in the post-modern thinking, which according to most researchers in the history of philosophy, began with the social revolution at the end of the 1960s. The most important "unbelief" of post-modernism is that there is no unchanging, ultimate or absolute truth. "Modernists did not believe the Bible is true. Post-modernists have cast out the category of truth altogether.

As Pointed out by Vogel, there can be no doubt about the difficulties we face when attempting to minister to the new mind. It is a formidable obstacle to biblical truth, especially because it rejects absolutes, while being unable to evaluate and integrate information into a whole concept.

Vogel (2000) has suggested the following guidelines (and some have been summarised by the presenter as follows):

1. The information overload has to be reduced, so that the new mind can reflect on its ideas and actions. We should persevere in demonstrating how consistent biblical lifestyles can help change the mind. Our challenge must be that the

creator has given His children, for their own good, the ability to think, to judge and to evaluate. With sincerest humility, we must guard Jesus' truth zealously and transmit it without adulteration to our generation and to generations that may follow. We can do this only as the Holy Spirit leads us personally into a deepening understanding of that truth.

2. From earliest days, we should teach our children that what is true is more important than what is relevant. We should also tell them that truth may not appear relevant at first sight but will reveal its relevance to the honest seeker.
3. The faulty logic and inconsistency of the new mind should never intimidate us.
4. In the face of today's pervasive relativistic philosophy, which either denies the existence of absolute truth or questions humankind's ability to grasp it, we should not be afraid to declare with humble boldness that claim to exclusive and absolute truth that rests on the divine revelation in the person of Jesus Christ and His inspired word. We should acknowledge, of course, that achieving this knowledge depends on adopting sound principles of interpretation.
5. The new mind or any mind for that matter should not be the starting point of our theology or practical living. Sometimes it seems that we have sought to win the favour of young adults at all cost, even the cost of truth.

CONCLUSION

Reasoning has played a major role in the development of science and technology. Without reasoning, the stage in which the world has reached in development wouldn't have been possible. However, it is important to realise that not everything can be explained by human reasoning. This is because God's knowledge is infinitely broader and deeper than ours; and the mathematics enterprise is an ongoing exploration of one segment of reality. Besides, logic has its limitations. As pointed out by the contemporary Harvard University logician, Willard Van Quine, relations of synonymy cannot be fully determined by empirical means. In addition, the "incompleteness" theorems proved in 1931 by Kurt Gödel (and their various consequences and extensions) are enough reasons why the mathematician should seek truth with "humility". The best way we can deal with issues (biblical or scientific) that we cannot explain by human reason is to suspend judgement and keep studying them prayerfully.

"Moreover, if all truth is God's truth and truth is one, then God does not contradict himself and in the final analysis there will be no conflict between the truth taught in scriptures and truth available from other sources" (Holmes, 1975). This fact should encourage us to teach the Lord's truth as it is to our generation and to generations that may follow. "We can do this only as the Holy Spirit leads us personally into a deepening understanding of that truth. For it is only as the truth enlightens our minds, possess our heart, and is incarnated in our life that we are enabled to stand rocklike in the midst of the truth-denying, truth-adulterating currents of our day. While pointing out where a position deviates from the biblical norm, we must guard against passing sentences on the motives of those who do not agree with us" (Vogel, 2000).

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