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# Certainty, Infinity, and Impossibility

# Implications for Faith and Learning

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*Abstract:* A chance remark to a collegue about the need for a course dealing with the philosophical and spiritual implications of notions that are fundamental to logic, mathematics and computer science led to the creation of a course for majors and nonmajors. The course would focuses on some of the fundamental limits of language and logic that are part of western civilization's rational heritage. The course topics include the finite and the infinite, the relationship between language, truth, and proof, computability including determinism and nondeterminism and the limits of computation, and Gödel's incompleteness theorem. This paper provides an overview of the course *that resulted* from that remark.

**Keywords and Phrases:** The Pythagoreans and the irrationality of $\sqrt{2}$, orders of infinity, Cantor's diagonalization argument, $|R|=|R^n|$; n=2, 3, ..., finite describability, language and meaning, proof, truth, logics, consistency, soundness, completeness, independence, effective descriptions, illusions, properties of n-dimensional space, impossible constructions, independent properties, computable and non-computable functions, computational complexity, determinism, nondeterminism, decidable, Church-Turing Thesis, Gödel numbering, Halting problem, incompleteness.

# Introduction

A chance remark to a collegue: "We need a course titled Infinity and Impossibility" growing out of mutual interests in the foundational concepts in mathematics, logic, and computation and their implications for faith led to the creation of a course which focuses on some of the fundamental limits of language and logic that are part of western civilization's rational heritage. This paper reports on the design of that course.

If you take as rational approach to life as possible, making logic your guide, what are the consequences? You will encounter hard limits on what you can do. You will be forced to conclude that humans are machines. What becomes clear is that in order to be human in the fullest most attractive sense, one must be willing to live with incompleteness.. To be fully Christian, one must include a spiritual dimension that, in contrast with rationality is mystical. This course is designed to help students develop a world view that is based on an understanding of the limits of rationality and a recognition of the role that subjectivity, spirituality plays in all religion.

The software engineer must build a bridge between the precise mathematical world of the computer and the vague, incomplete, evolving, and possibly contradictory world of customers who often do not know what is possible or even what they want. Communication with the customer involves the explicit use of communication theories and literary theories (including hermeneutical methods). The mathematical side is constrained by the limits of logic and computational theory. The interaction between these worlds is not unlike that which we find in the western world where the Christian lives in two worlds, the sacred and the secular, each with its own set of values and the Christian must navigate between the two. However, that western world has changed. Due to globalization, the western world is confronted with a level of diversity in thought, culture, and religion for which it is largely unprepared.

The issues of language, logic, and meaning form a unifying theme in exploring the worlds of the customer, mathematics, the sacred, the secular, and the cultural. The similarities in the languages used for doctrine, logic, mathematics, and computation raise intriguing questions concerning the implications of the limits of reason and objective reality that under-lie rational western civilization and our conception of the spiritual world.

The rationality and truth of mathematics, the effectiveness of science, and the transcendent nature of religion are integral parts of our western culture.. These disciplines both complement and challenge with each other. There are similarities in the languages used for doctrine, logic, and mathematics. These similiarities raise intriguing questions. For some questions, there are answers, for others, the answers are not as forthcoming. This course reflects my personal journey and my sense that others may benefit from considering these issues, the answers that I have found, and the approaches that I have used.

The technical ideas presented in this course are standard material in the foundations of mathematics and computer science. The philosophical implications of some of these ideas are standard material in philosophy. However, the application these ideas to issues of faith is rarely available. In any case, this material is not readily available in courses accessible to a wide variety of students on Adventist campuses. A course with this material serves two purposes. For mathematics and computer science majors, it is an introduction to some of the social issues related to their disciplines. For other majors, it provides an accessible introduction to the role, value and limits of reason.

Following a preliminary unit on sets, relations, and functions to provide a common language, the course consists of three units with the following titles:

1. In Touch with the Infinite: The Finite, the Infinite, and the Transfinite
   Key ideas: The Pythagoreans and the irrationality of $\sqrt{2}$, orders of infinities, Cantor's diagonalization argument, $|R|=|R^n|$; n=2, 3, ..., describability.
2. The Search for Certainty: Logic, Proof, and Truth
   Key ideas: language and meaning, proof, truth, logics, consistency, soundness, completeness, independence, finite and effective descriptions.
3. Its not Just Hard, it's Impossible
   *Key ideas*: illusions, properties of n-dimensional space, impossible constructions, independent properties, computable and non-computable functions, determinism and nondeterminism, Gödel numbering, diagonalization, the Halting problem, Incompleteness.

Each unit consists of lectures, discussion and essay questions, and a reading list. The lectures approach the technical content from historical and intuitive points of view. The discussion and essay questions are designed to engage the interests of students from a wide variety of background in productive discovery and discussion. The reading list provides a source for required book reports and opportunities for further exploration on technical, historical, and philosophical levels. The following sections provide an overview of the lecture content and a sampling of the discussion and essay questions.

Sections 2, 3, 4 are the core of this paper and are a slight rearrangement of the topics in the three units listed above. Each section introduces a topic, presents some technical details, considers the implications of the topic in the area of faith and learning, and provides a sampling of the sort of discussion and essay questions contained in the class.

The relationship between the finite and the infinite plays a central role in understanding the limits of reason and computation. Section 2 is a discussion of the relation between the finite and the infinite. Section 3 is a discussion of language, truth, and proof. It is divided into two subsections. Subsection 3.1 is a discussion of logic (syntax) including Gödel's incompleteness theorem. Subsection 3.2 is a discussion of semantics. Section 4 is a discussion of the nature of computation and determinism versis nondeterminism. Section 5 is the summary and conclusions. Section 6 contains the references.

## Numbers: The Finite and the Infinite

Numbers and numerology have played and continue to play a significant role in many cultures. The Pythagoreans (c. 500 BC) were a mystical Greek brotherhood that believed that reality at its deepest level is mathematical in nature and based on the natural numbers or ratios of the natural numbers. Their irrational belief was thrown into disarray with the discovery of the irrational numbers (specifically that $\sqrt{2}$ is irrational).

The infinite was equally mysterious in early societies, Zeno's paradoxes are among the best known. The story of the race between the tortoise and the hare illustrates the confusing nature of the infinite. The tortoise was given a head start. The hare could never catch up because each time the hare reached the point where the tortoise had been, the tortoise had moved on a little farther. The paradox being, how could an infinite number of sums equal a finite number?

The notion of the infinite has been controversial. Throughout the history of mathematics, some have rejected the notion on philosophical grounds while others have enthusiastically embraced it for pragmatic reasons. In the late 1800s, Georg Cantor developed set theory for reasoning about the infinite. One of the remarkable ideas to emerge is that there is an infinite hierarchy of infinite cardinal numbers designated $\aleph_0, \aleph_1, \cdots$ each infinitely larger than the previous one. Today, the mathematical community is largely pragmatic in their approach to the infinite. Making use of it where it advances mathematical knowledge and avoiding it in finite domains like computation where its use prevents the construction a computer program.

While the words *infinite* and *finite* hardly appear in the Bible, they are popularly employed in sermons and religious writings to describe the difference between the Creator and the created. Their imprecise and ambiguous use in the religious community contrasts sharply with their precise definition and use in the scientific and technical community. This diference in use is a potential source of confusion.

The key ideas of this section are the observations that, given a finite alphabet, any language based on that alphabet will consist of at most a countably infinite set of sentences. With those sentences, there are an uncountably infinite number of sets of those sentences.

## The technical details

Without loss of generality and for simplicity, the following discussion is based on the set {0, 1}. The conclusions apply regardless of the natural language and alphabet.

Let $\Sigma$ be the set {0, 1}, a finite set called the alphabet. Let $\Sigma^*$ be the set consisting of the empty string and all of the finite strings composed of the symbols of the alphabet. It is not hard to see that $\Sigma^*$ is the set of the natural numbers represented in binary form. Any set of the size of $\Sigma^*$ is said to be countable. The set of natural numbers, the set of integers, and the set of rational numbers are all countable.

It is a feature of infinite sets that they can be placed in one-to-one correspondence with a proper subset of itself so for example, the natural numbers can be placed in one-to-one correspondence with their squares.

n: 0 1 2 3 4 5 ...

$n^2$: 0 1 4 9 16 25 ...

Another observation concerning the infinite subsets of infinite sets is that the infinite subsets of the natural numbers are unbounded in the sense that for any natural number in the subset, there is always a larger natural number in the subset. Dense infinite sets (like the rationals and the reals) have bounded infinite subsets for example, the interval (0, 1) is an infinite subset of the real numbers. It almost seems paradoxical that a set of finite length (the interval 0, 1) contains an infinite number of elements.

The power set of $\Sigma^*$, $\wp(\Sigma^*)$, is the set consisting of all subsets of $\Sigma^*$. It is of infinite size and it contains both finite and infinite sets as its elements. The infinite sets, $\Sigma^*$, the set of even numbers, and the set of prime numbers, are elements of the power set. This set is uncountable and can be placed in one-to-one correspondence with the real numbers. Since it is uncountable, it is a different order of infinity from $\Sigma^*$ and the set of natural numbers. The proof of this assertion by means of a diagonal argument is beyond the scope of this paper but is easily understood.

This process can continue creating an infinite sequence of ever larger infinite sets. This countably infinite sequence of infinite numbers is designated $\aleph_0$, $\aleph_1, \cdots$; where $\aleph$ is aleph, the first letter of the Hebrew alphabet. An interesting problem is the generalized continuum problem which is whether there is an infinite set X between each infinite set S and its power set, $\wp(S)$.

It is easy to show that the elements of $\Sigma^*$ correspond to all possible descriptions in natural languages. Since the set $\wp(\Sigma^*)$ is of a higher order of infinity than the set $\Sigma^*$, there must be elements of $\wp(\Sigma^*)$ that are not describable. That is, not all descriptions in $\Sigma^*$ correspond to elements of $\wp(\Sigma^*)$ and not elements of $\wp(\Sigma^*)$ have descriptions.

## What does it mean?

If human beings and the natural universe are finite, then either should be describable by a finite string. As indicated earlier, $\Sigma^*$ corresponds to the set of natural numbers and is an infinite set. The subsets $\wp(\Sigma^*)$ are all of the unary properties of the natural numbers. For example one element of the set is the set of even numbers and another is the set of prime numbers each of which are sets of infinite size. The set of natural numbers is completely describable but not all of the unary properties of the natural numbers are describable since the number of finite descriptions (the size of $\Sigma^*$) corresponds to $\aleph_0$, and the number of properties (the size of $\wp(\Sigma^*)$) corresponds to $\aleph_1$with $\aleph_0 < \aleph_1$. For emphasis, the infinitude of the natural numbers is $\aleph_0$ while the infinitude of the real numbers is $\aleph_1$. The proof of this assertion is beyond the scope of this paper but is easily understood.

What does it mean for God to be infinite and humans finite? The simplest interpretation is that there is an unbridgable gap between them. Using reasoning by analogy, the hierarchy of infinities suggests that the best we can do is to ask whether God describable. I will take up this question in a later section.

The set theory articulated by Georg Cantor led to paradoxes (actually antinomies) such as the following:

- The set that contains all sets that do not contain themselves. Does it contain itself? Paradox: it does if it doesn't and doesn't if it does.
- There is a village with only one barber. The barber, a man, shaves all the men that do not shave themselves. Who shaves the barber? Paradox: He does if he doesn't. He doesn't if he does.
- The Liar: Assume that by "liar" is meant a person who *never* tells the truth. Cretan philosopher Epimenides (sixth century B.C.): All Cretans are liars.

These paradoxes, among other things, prompted David Hilbert, at the turn of the 19th century, to publish a list of unsolved problems as a challenge to the mathematical community. Among the problems were the challenge to prove that arithmetic is consistent and a challenge to define the notion of an effective method. Both of these problems revolve around the distinction of the finite and the infinite and these problems are considered in the next two sections. Logic and the "proof of consistency" is the subject of the next section.

The following are a sample of the discussion and essay questions on this topic:

- The belief system of the Pythagoreans was based initially on incomplete information and they took a position which later turned out to be contradictory. What are the lessons for us? How should we deal with contradiction? Discuss: some say that fundamentalism resolves paradox by accepting only the position of faith and is intolerant of those who hold a contrary view. If contradiction is inevitable in human cultures, what mechanisms are available to cope with the contradictions. What should be a basis for a belief system?
- Mathematically, there is an infinite hierarchy of infinities. Does this have any implication for the structure of reality? Often the attribute of "infinite" is ascribed to the deity. What should we understand by the attribute? Often the attribute of finiteness is ascribed to humans. What should we understand by the attribute?
- The paradoxes in set theory arose in part because of the use of informal language. They were resolved through careful formalization however, most people find such formalism unacceptable. Are humans constrained to live in an inconsistent state? What are possible solutions?

# Logic (Language, Domain, & Method)

The subject of logic naturally separates into two parts - proof theory and model theory. Proof theory is the study of the syntax of the language of logic and the methods of proof - a game of symbol manipulation. Model theory is the study of semantics or meaning of the sentences of the language - the construction of meaning. Proof theory is the subject of the first subsection and model theory the subject of the second subsection.

## The rules of the game and the limits

*In the beginning was the word.* John 1:1; *Prove all things.* 1 Thessalonians 5:21

David Hilbert's challenge to prove the consistency of arithmetic and the discovery of paradoxes in Georg Cantor's set theory resulted in a flurry of activity among mathematicians. A number of remarkable results followed. Among them, Kurt Gödel's famous incompleteness theorem which demonstrated that the consistency of arithmetic could not be established by the formal means envisioned by David Hilbert. This theorem may be stated in several ways (Wang 1996) among which are:

- Mathematics is inexhaustible.
- Any consistent formal theory of mathematics must contain undecidable propositions.
- No theorem-proving computer (or program) can prove all and only the true propositions of mathematics.
- No formal system of mathematics can be both consistent and complete.
- Mathematics is mechanically (or algorithmically) inexhaustible (or incompleteable).

Gödel himself observed:

> A completely unfree society (i.e., one proceeding in everything by strict rules of "conformity") will, in its behavior, be either inconsistent or incomplete, i.e., unable to solve certain problems, perhaps of vital importance. Both, of course, may jeopardize its survival in a difficult situation. A similar remark would also apply to individual human beings. - from Wang, H. *A Logical Journey: from Gödel to Philosophy.* The MIT Press. 1996.

While Gödel's result is perhaps the most significant mathematical insight of the 20th century, it is one of several observations that need to be understood in the context of formalizing the reasoning process.

**The technical details**

A formal system consists of a language, a structure, a correspondence between the language and the structure and some inference rules that describe how to construct a new sentence from zero or more sentences. The is used to describe the properties of the structure and the inference rules support reasoning about the structure. Let $\Sigma$ be a possibly countably infinite set of symbols and $L(\Sigma)$ be a language (a set of sentences or strings), defined using the symbols in $\Sigma$, for $M=\langle S, I\rangle$ a *structure* for the language where $S$ is a non-empty set and $I$ is correspondence between the sentences of the language and $S$. For a sentence, $\Phi$, of the language, $M$ is a *model* of $\Phi$, written $M \models \Phi$, if the sentence is true in $M$ i.e., $I(\Phi) \in S$. A *theory* is a set of sentences and for a given theory $T$, $M \models T$ denotes that $M \models \Phi$ for each sentence $\Phi$ in $T$. For now, I will ignore the inference rules.

*Multivalued logics.* Aristotelian logic, with its two values (true and false), is the sort of logic that commonly comes to mmind when the subject of logic is raised. In two valued logic, the correspondence mapping maps sentences to the set {false, true}. In multivalued logics the mapping may be to any number of values. The Likert scale (0, 1, 2, 3, 4, 5) used to indicate preferences is an example. In infinite valued or continuous logics, the set of truth values is infinite and are often represented by numbers in the unit interval, [0, 1]. Reasoning with probabilities is an example.

*Finite and effective: sentences, proofs, theorem.* For any given formal system, there are three general questions of interest.

1. Is an arbitrary string in the language a sentence?
2. Is a finite sequence of sentences a proof?
3. Is a given sentence a theorem?

The first two questions are answered by straight forward reductionist techniques. The first question corresponds to determining whether an arbitrary sequence of words is a sentence by applying the rules of grammar for the language. The second question is a matter of recognizing whether each element in the sequence is either an instance of sentence in the theory or follows from earlier elements by a rule of inference. The third question is fundamentally different. The answer to the third question is not determined by simply looking at the parts of the sentence. Gödel's incompleteness theorem tells us that we cannot in general answer this question.

*Consistency.* Consistency requires that a system be free from contradiction, that is, both a statement and its negation may not be both true. Where a language does not include negation, consistency is defined as not all strings are in the language.

*Sound rules of inference.* Rules of inference (deduction) facilitate the extension of a theory from the small set of axioms to new knowledge (theorems). The rules of deduction must be sound (truth preserving) so that the theorems add true knowledge. A common rule of deduction is *modus ponens*, from A and $A \rightarrow B$ infer B.

*Gödel's incompleteness theorem.* Gödel found a way to turn logical expressions into numbers so that a question about logic could be turned into a question about numbers. Smullyan describes it as follows:

> He showed that for a large class of mathematical systems, one can assign to each sentence a number called the *Gödel number* of the sentence, and then construct a sentence $X$ asserting that a certain number $n$ has the property of being the Gödel number of a sentence that is *not* provable in the system, but this number $n$ is the Gödel number of the very sentence $X$ itself! And so $X$ is true if and only if its Gödel number is not the Gödel number of a sentences provable in the system -- in other words, $X$ is true if and only if it is not provable in the system. This means that either $X$ is true but not provable in the system, or $X$ is false (not

true) but provable in the system. Under the assumption that the system is *correct* in that only true sentences are provable, the second alternative is out, hence the sentence is true but not provable in the system.

*Independent axioms, multiple models and the Pythagoreans revisited.* The parallel postulate of plane Euclidean geometry states that through a point not on a line, there is on and only one line parallel to the given line. The parallel postulate is independent of the other postulates of geometry. On the surface of a sphere, lines are interpreted as great circles and there are no parallel lines. On other surfaces, there are multiple parallel lines. These alternate models of the other postulates demonstrate that the parallel postulate is independent of the other postulates.

The recognition of the independence of the parallel postulate and the discovery of non-Euclidean geometries is every bit as significant as the Copernican revolution in astronomy and the Darwinian revolution in biology. The key concept here is that any theory has multiple realizations and that any subset of a set of independent axioms leads to a consistent theory in which the other axioms are false. Attempts to create an absolute theology are bound to fail.

One of the surprising consequences of Gödel's incompleteness theorem is that there are non-elementarily equivalent, countable models of formal arithmetic. Such models are called *non-standard models of arithmetic.* To emphasize the first sentence again, these are *countable* models meaning that even though the real numbers are uncountable, only a countable model (thus equivalent to the natural numbers) is required. The Pythagoreans may have been right after all.

*Theory construction.* In mathematics, the desired design goals include a small set of independent axioms which facilitate proofs of the properties of interest of the system of interest. Likewise in science, the desired design goals for a scientific theory include a small theory of independent concepts. In software engineering, the desired design goals include making program robust which is in part achieved by making the program modules independent as possible.

Independence is essential to make theories robust. In the case of science, this permits the piecemeal revision of theories as new information is encountered that perhaps contradicts a portion of the theory. If the axioms are not independent then should one of the axioms be contradicted, then the whole theory collapses and must be thrown out. In software, independence in modules simplifies the maintenance and evolution of the software improving its usefulness and lifespan.

**What does it mean?**

I find the formal approach of structure, language, correspondence, theory, and inference rules to be a useful tool for organizing knowledge. For example, a collection of doctrines constitutes a theory expressed in a language and given meaning by their relation to a text. A text, in turn, is a theory expressed in a language and given meaning by its relation to the natural and supernatural worlds. From Gödel's theorem it is clear that the choice of a set of axioms for the theory will have significant effect on what we can learn. It is highly desirable that the axioms be independent of each other. This same principle applies to doctrines. If doctrines are not independent, then the religion based on them cannot be robust and collapses if a single contradiction is uncovered.

While Gödel's incompleteness theorem was proved in the context of arithmetic, it holds for most any system of sufficient complexity that is formalizable. Restating it in the more general general terms:

- Interesting theories are inexhaustible.
- Any consistent interesting theory must contain undecidable propositions.
- No theorem-proving computer (or program) can prove all and only the true propositions of an interesting theory.
- No interesting theory can be both consistent and complete.
- Interesting theories are mechanically (or algorithmically) inexhaustible (or incompleteable).

Gödel's own observation can be extended to religious doctrine as well:

> An entirely rational religion, with a completely logical collection of doctrines, will be either inconsistent or incomplete, i.e., unable to solve certain problems, perhaps of vital importance. Both, of course, may jeopardize its survival in a difficult situation. A similar remark would also apply to individual human beings.

The physicist John Borrow observed:

> If a *religion* is defined to be a system of thought which contains unprovable statements, then Gödel has taught us that not only is mathematics a religion, but it is the only religion able to prove itself to be one. -- physicist John Barrow

Some observations of my own include:

- Interesting theories will contain islands of truth that are unreachable from a particular set of axioms.
- Complete theories are useful to the engineer but of little interest to the mathematician.
- The theorem imposes real limits on what we can know from a given viewpoint, which is a strong argument for including a wide range of viewpoints in making important decisions.

Mathematics begins with an infinite domain and a complete description of that domain. Then mathematicians explore all the logical consequences of that description. Since there are an infinite number of relations available for study, mathematicians will be happily occupied for some time without further contact with the domain.

Suppose scientific research stopped with a particular generation of scientists, say Newton. There would be a lot of questions we could not answer no matter how hard we concentrated on Newton's theory. That is because scientists do not have a complete description of nature and progress in science requires contact with the stuff of science, nature.

Suppose theology stopped with a particular collection of doctrines. There would be a lot of theological questions that could not be answered no matter how hard theologians concentrated on that particular collection of doctrines. Progress in religion occurs by contact with the stuff of religion, the source texts and the mystical union with the divine. For this reason and for the fact that theologians have constructed incompatible sets of doctrines, each claimed to be a description of the revealed text, theology bears a stronger resemblance to science than to mathematics. In this respect, Bible study resembles experimental science while doctrinal studies resembles theoretical science. Both approaches are necessary to maximize spiritual growth.

The view of logic presented in this section is almost entirely syntactical - what symbols are used and how they are manipulated. In the next section the attention shifts to semantics - what the symbols mean. Formally it is what is called model theory but my focus will be entirely on the construction of the correspondence between the language and the structure. To some extent it is what we might call *applied logic.* The motivating example comes from software engineering which begins with the need of a customer and ends with the delivery of a software product. The product may be viewed as giving meaning to the customer's need.

## The construction of meaning

> *And the Word became flesh.* John 1:1; *Howbeit when he, the Spirit of truth, is come, he will guide you into all truth.* John 16:13

Alfred Tarski established model theory by making explicit what mathematicians knew implicitly that their mathematical expressions represented or corresponded to various set theoretic structures. In his formulation, the truth of a sentence is determined by virture of its precise mapping to a structure. That is, sentences in a language are deemed meaningful in a domain of discourse by virture of a relationship between the language and the domain of interest.

There are three general approaches to defining the meaning - coherence, correspondence, and a social theory.

The *coherence* theory requires that the truth of any (true) proposition consists in its coherence with some specified set of propositions. The relation between propositions and their truth conditions is coherence (consistency). The truth conditions of propositions consist in other propositions. Well written fiction posesses coherence and may be independent of reality. A theory that is carefully constructed, coherent, and consistent does not necessarily contrubute to a better understanding of our universe. For such a theory to make a contribution, there must be some correspondence, real or imagined, with the real world.

The *correspondence* theory requires that the truth of any (true) proposition consists in its correspondence with a feature of the world. The relation between propositions and their truth conditions is correspondence. The truth conditions of propositions are objective features of the world. The correspondences in mathematics are accurate while the correspondences in scientific theories are vague within some limits of precision.

The *social* theory requires that the truth of any (true) proposition consists in its designation as truth by some social group. The relation between propositions and their truth conditions is an ongoing achievement (work) of some social group. The truth conditions are situated, local, contingent, embodied, vague, and open. Most social structures, family, government, religion, and even software, are constructed by a social processes. Social processes are necessary when information is vague, incomplete, evolving, and possibly contradictory.

Coherence in its simplest form, is freedom from contradiction which was covered in subsection 3.1. The construction of meaning meaning involves three considerations - a domain of interest, a language used to describe and reason about the domain of interest, and a correspondence between the language and the domain. The software engineer uses natural and artificial languages, the computer and the application domain, and constructs a variety of correspondences in the course of producing a software product.

**The technical details**

*Correspondence theory of truth.* At the heart of logic, as used in mathematics, is the correspondence theory of truth (or meaning) as articulated by Alfred Tarski. The basic idea is as follows. There is a structure of interest for example, the natural numbers. A language is developed to describe the structure (the language of elementary arithmetic) with a formal and precise correspondence constructed between the language and the structure. The axioms written in the language are required to be accurate and precise description of the structure. In the case of the natural numbers, the axioms of Peano are typical of the approach. This precise correspondence insures that the theory developed from the axioms will be consistent. Table 1 is a succinct depiction of the correspondence theory of truth.

**Table 1:** Correspondence theory of truth

| *Representing system (meta level)* | *Meaning (semantic mapping)* | *Represented system (object level)* |
|---|---|---|
| a, ... | → | a, ... |
| Formal<br>abstract<br>proof<br>language | 1 to 1<br>(accurate and precise) | Informal<br>concrete<br>truth<br>structure |
| | *Examples*: | |
| assembly instructions<br>road map<br>mathematical theories<br>software requirements | → | componetized product<br><br>road system<br>mathematical structure<br>software product |

*The social theory of truth (Goguen).* In sharp contrast to the well-defined problem in mathematics are the vague, contradictory, and dynamically changing problems encountered by software engineers. The most difficult challenge for software engineers is that of elicitating a clear statement of the problem from the set of stakeholders and how to resolve the competing goals and subtle political issues that can interfere with the acceptance and use of the final product. I see the problem of requirements elicitation as one of constructing a language to describe the problem domain which is the structure of interest. The final software product becomes an alternate realization of the problem domain (the structure). The important observation here is that while there is a precise mapping from the software requirements to the software product, the mapping from the requirements to the problem domain of the stakeholders is not precise. In both cases, the issue is one of constructing meaning. The "truth" of the requirements depends on the mapping to both the problem domain and the software product. Because the problem domain is both complex and dynamic, the engineering of software must be an ongoing project.

Joseph Goguen and others looked at using various methods ranging from methods of literary analysis to communication theories to solve these problems. He proposed the idea of a *social information system* to formalize his understanding of the methods and issues involved.

The identifying characteristics of a social information system are the following:

- A dynamic domain (the object world).
- A configuration of signs (a dynamic meta world).
- A social group embedded in the dynamic domain which is responsible for maintaining a mapping from the meta world to the object world.
- The mapping
  - involves contradictory positions and
  - myths rooted in historical events that persist in the face of objective counter evidence.

An *item of information* is an interpretation of a configuration of signs for which members of some social group are accountable.

*Meaning* is an ongoing achievement of some social group; it takes work to interpret configurations of signs, and this work necessarily occurs in some particular context, including a particular time, place, and group. The meaning of an item of information consists of the relations of accountability that are attached to it in that context, and the narratives in which it is embedded. Information is tied to a particular, concrete situation and a particular social group with the following consequences.

1. *Situated.* Information can only be understood in relation to the particular, concrete situation in which it actually occurs.
2. *Local.* Interpretations are constructed in some particular context, including a particular time, place, and group.
3. *Emergent.* Information can only be understood through the ongoing interactions among members of a group.
4. *Contingent.* The interpretation of information depends on the current situation which may include the current interpretation of prior events.
5. *Embodied.* Information is tied to bodies in particular physical situations, so that the particular way that bodies are embedded in as situation may be essential to some interpretations.
6. *Vague.* Information is only elaborated to the degree that it is useful to do so; the rest is left grounded in tacit knowledge.
7. *Open.* Information is open to revision in the light of further analysis and further events.

Groups, values, and information are *coemergent*, in the sense that each produces and sustains the other; values exist because they are shared and communicated by groups; and information exists because groups share values in a dynamic world.

Table 2 puts the social theory of information in the format used for the correspondence theory of truth. Since the mapping between the representing system and the represented system is dynamic in a number of dimensions, the mapping may be described as an ongoing achievement (work) of some social group. The mapping is situated, local, contingent, embodied, vague, and open.

Several things combine to make problems in social environments difficult and often wicked. Human societies inhabit a dynamic world which is too complex for an individual or even a society to formalize in a theory and validate. As a consequence, each individual develops a partial theory that may differ from other individuals due to differing experiences and roles. Further, human societies formulate shared theories (cultures) that are transmitted to succeeding generations. Given the dynamic nature of the world, these cultures do not necessarily map to the next state of the world. The partial mappings and the accumulation of differences combine to create the situations that result in wicked problems. In this formulation, wicked problems are the consequence of incomplete and inconsistent mappings produced over time by individuals and social groups.

**Table 2**: Social theory of information

| *Representing system* *(meta world)* (a configuration of signs) | vague mapping 1→n | *Represented system* *(object world)* |
|---|---|---|
| Formal abstract language | An ongoing achievement (work) of some social group. The mapping is situated, local, contingent, embodied, vague, and open | Particular context including time, place, group, relations of accountability, and narratives (text, video, sound recordings) |
| *Examples* | | |
| Corporate culture | Management and labor | Products, market place, consumers |
| Church doctrines | A particular church body | Scriptures and a group |
| Software requirements | Requirements engineers and stakeholders (must be verifiable) | Needs and goals - the high-level objectives of the system, and the users/stakeholders. |

By way of contrast, mathematicians concentrate on small static worlds which facilitate the validation of mathematical theories. Scientists, in the natural sciences, while concentrating on dynamic worlds, limit their study to systems that do not develop cultures and use repeated experimentation to validate their theories.

*Other logics.* One of the important contributions of computer science in particular, research in artificial intelligence, to logic is the increased attention paid to non-classical logics. Classical logic is monotonic. Given a set of valid axioms, each theorem adds new truth to the system. A theorem once proved, remains true forever. Such a logic is a natural match for mathematics but is entirely unsuitable for science where theories are subject to revision and certainly for real life. The Tarskian approach to semantics suggests that language and inference rules should be constructed to facilitate reasoning about the semantic domain. In contrast with classical logic, temporal logic reflects the fact that the statement "Today is Tuesday" is true only on Tuesdays; infinite valued logic reflects the fact that the words "happy", "sad", "hot", and "cold" are not true or false but partly true and partly false; default logic permits statements to be tentative and dependent on further information. Logics that permit revision are call non-monotonic logics and such logics are an active area of research particulary useful in the artificial intelligence systems employed in robotics.

**What does it mean?**

Of the three theories of meaning, the correspondence theory is the only one that is objective, verifiable, and suitable for verification and so it is the only acceptable theory for use in mathematics where objectivity and precision is required. Since theoretical mathematics is about artificial worlds rather than the natural world it is easy to construct an accurate correspondence. By way of contrast, in software engineering, which occupies two worlds -- the vague, incomplete, and changing world of the customer and the formal, mathematical, and artificial world of the computer, the construction of an accurate correspondence is difficult. It is clear that the formal, artificial world of logic, while suited for the world of the computer, is not suitable for communication with the customer. Thus the methodology of the on going process of the social theory of meaning is absolutely necessary in the world of the software engineer. The correspondences for religious doctrine derive are derived through social processes. The subjectivity and unreliability of social processes is demonstrate by the fact that after two thousand years of study, Christian theologians are not about to agree even on basic church doctrine.

The nature of the correspondence between a theory and reality is at the heart of the debate over science and religion. Is a religious theory (set of doctrines) any less real than a scientific theory? For every structure there is an infinite number of theories that correspond to the structure i.e., each theory is true in the structure. Two individuals watching the sky together for a year, collecting the same data, could very well come to two different conclusions, one that the sun travels around the earth, the other that the earth travels around the sun. It is possible to have two incompatible theories that are true with respect to the same set of facts.

In wicked problems, it is not clear from the beginning what theproblem is and thus, what a solution is. Therefore, finding a solution requires in addition, finding out what the real problem is. Solving and specifying the problem develop in parallel and drive each other. Wicked problems usually have vague, ambigous, and contradictory requirements which often change during the search for a solution. Even when the problem is finally understood, the solution is often unknown and difficult to recognize and those that are found are often such that they still could be improved and it is up to the problem solver to decide when enough is enough.

As much as we may want to emulate mathematics in the development and justification of religious doctrine, we are bound to fail. All religious doctrine is established and maintained through a social methodology.

Social groups and organizations invest considerable time and energy attempting, through educational programs, to reduce the natural conflicts that develop due to naturally emerging differences between individuals and subgroups.

This section considered the results of David Hilbert's challenge to prove the consistency of arithmetic. Following the discussion and essay questions, the next section considers his challenge to define the notion of an effective method.

### Discussion and essay questions for proof theory and model theory

The following are a sample of the discussion and essay questions on this topic:

- Compare and contrast the correspondence (or social) theory of meaning/truth with that of a literary theory/hermeneutical method, a communication theory, or the scientific method.
- Pick a work of art (painting, sculpture, music, etc) and describe how your aesthetic judgement constructs meaning.
- Is a religious theory (set of doctrines) any less real than a scientific theory?
- Under what circumstances is it possible to resolve distinct and incompatible world views?
- Determine whether some doctrine is constructed by deduction, induction, a social group, or revelation.
- It has been observed that if any element of a system is false, then the whole system collapses. Under what conditions is this observation true and under what conditions is it false?
- Gödel's incompleteness theorem tells us that the systems we build can't be both complete and correct and that the systems we build cannot determine their own semantics. Under what circumstances can doctrines be both complete and correct? Under what circumstances can the Bible be its own interpreter?
- Some suggest that monotheism is coherent with two valued logic, absolutism, intolerance, and authoritarianism. Show how to construct a monotheistic system that is not coherent with absolutism, intolerance, or authoritarianism.
- Do our mystical insights arise from supernatural inspiration, rational subconscious processes, or rational attempts to reason in the face of incomplete, inexact, and/or contradictory information? Is all religion ultimately mystical and non-logical? Can we be completely rational or must we also be mystical?
- Given that there is wide disagreement between religions and even among Christian theologians on basic doctrine, on what basis do religions have a mandate to prescribe morality or claim absolute truth? If you can *prove it*, why don't they believe you? Is it possible to create an absolute theology?

## Computation

David Hilbert's tenth problem was to devise an algorithm that tests whether a polynomial has an integral root i.e., find a process which determines the result in a finite number of operations. If such a process were discovered, it would contribute little to our understanding of the nature of a process. If in fact, no such process exists, then in order to show that no such process exists, we require a universally agreeable definition of process to be used in the proof. Alfred Tarski, Kurt Gödel, Alan Turing, Emil Post, and Alonzo Church, among others, worked to produce an acceptable definition.

*For some things, there is no procedure,*
*no description, or*
*even a name.*

*Though these are truly real,*
*only what is finite can be known.*

*They are all there*
*but we see only a vague shadowy outline*
*and are able only to guess at the hidden relations.*

*But, this is where it begins.*

Tao Tê Ching, chapter 1.

## The technical details

*Effective computability and the Church-Turing thesis.* The many definitions of computability produced by a host of researchers have been proved to be equivalent. With apparently no alternative ideas left, Alonso Church proposed, as a thesis, that recursive functions, as the most elegant mathematical definition capturing this intuitive notion of an algorithm, are sufficient to define all computable functions. Since the Turing machine (Alan Turing's proposal) is the definition of choice among computer scientists, Church's thesis is often called the Church-Turing Thesis.

**Table 3:** The Church-Turing Thesis

| | | |
|---|---|---|
| *Intuitive notion of an algorithm* | equals | • Recursive functions (λ-calculus),<br>• Turing machines, or<br>• Unrestricted grammars |

The Church-Turing thesis received unexpected support from an apparently unrelated area. Noam Chomski, in his study of grammars for natural languages, developed a hierarchy of grammars . Each type of grammar in the hierarchy has been shown to be equivalent to a class of machines. Chomski's unrestricted grammars are equivalent to Turing machines. Therefore there is support for the Church-Turing thesis from three very different areas, recursive functions, Turing machines, and formal grammars.

**Table 4:** Grammar/Machine Hierarchy

| **Grammar** | **Machine** |
|---|---|
| Unrestricted grammars | Turing machines |
| Context-sensitive grammars | Linear bounded automata |
| Context-free grammars | Push-down automata |
| Linear grammars | Finite automata |

Computability theory addresses the questions: What can be computed? and What cannot be computed? We all ready know that there are only a countably infinite number of finite strings (things that could be used as names) while there are uncountably infinite number of sets (things that need names), therefore there are things that cannot be named.

*Turing Machines.* The Turning machine was proposed by Alan Turing in 1936. It consists of an *infinite tape* of cells each of which can contain a symbol, a *tape head* that reads and writes symbols on the tape and moves left and right along the tape, and a *finite state control.* The formal definition of a Turing machine is found in figure 1.

*Definition*: A finite input string is **recognized** by a Turing machine, if the Turing maachine starts in the start state with the read-write head at the left end of the tape and after a finite number of moves the read-write head has visited each cell of the input and enters an accepting state. Recognition means that the TM halts in an accepting state for elements of the language and either halts in a rejecting state or fails to halt for elements not in the language.

**Figure 1:** Defintion of a Turing Machine

TM = $\langle Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject} \rangle$

1. $Q$ is the finite set of states.
2. $\Sigma$ is the finite input alphabet not containing the special **blank** symbol.
3. $\Gamma$ is the finite tape alphabet, where $\{\textbf{blank}\} \cup \Sigma \subseteq \Gamma$.
4. $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function of the finite control.
5. $q_0 \in Q$ is the start state.
6. $q_{accept} \in Q$ is the accept state.
7. $q_{reject} \in Q$ is the reject state, where $q_{reject} \neq q_{accept}$

$\delta(q, a) = (r, b, D)$ - when the machine is in the state q and the head over tape symbol $a$, the machine writes the symbol $b$ replacing $a$, goes to state $r$, and moves the head one cell in the direction $D$.

*Definition*: A language is **Turing-recognizable**(recursively enumerable) if some Turing machine recognizes it.

*Definition*: A language is **Turing-decidable** (recursive) or simply *decidable* if some Turing machine decides it. Decidable means that the TM always halts in an accepting state for recognized strings and in a rejecting state for unacceptable strings.

**Table 5:** Relationship between computable and noncomputable functions

| | |
|---|---|
| *Noncomputable functions* (Turing machines that do not halt in an accepting state.) | This is the largest group of functions. |
| *Computable functions* (Turing machines that halt in an accepting state.) | Intractable functions (exponential complexity or worse) |
| | Tractable functions (polynomial complexity) |

*Universal Turing Machines*. One of the amazing properties of Turing machines is the existence of a universal Turing machine. The universal Turing machine is a Turing machine that can simulate the behavior of any another Turing machine i.e., given the description of another Turing machine it can behave as if it were that other Turing machine. Computers are examples of universal Turing machines.

**Universal machine**

A machine $U$ is said to be universal if for each machine $W$ there exists a string $w$ such that $U(wx) = W(x)$ for all strings $x$.

The acting profession is an example of universal machines for humans.

*Undecideable questions*. One of the inportant questions to ask about Turing machines is whether or not a Turing machine will halt for a given input. The is the famous Halting Problem. The answer is that while the answer is yes for some Turing machines and some input, the answer is no for Turing machines in general. This result is related to Gödel's incompleteness result in logic.

*Computational complexity 1*. The complexity (performance) of an algorithm is measured in how many steps it takes to compute a solution of a problem of size $n$. The complexity is expressed as a function of $n$. Table 6 illustrates this idea. Note the enormous increase in time for exponential functions. In general theorem proving is of exponential complexity. The implication is clear, that, in general, problems of exponential complexity are beyond the reach of humans. The class of problems that are solvable in at most polynomial time are considered to be tractable problems. This class of problems, P, are problems that are solved in polynomial time on deterministic Turing machines.

*Determinism vs. non-determinism andComputational complexity 2*. A nondeterministic Turing machine has a transition relation rather than a transition function. For example: $\delta(q,a) = \{(r_i, b_i, D_i), (r_j, b_j, D_j), \ldots\}$. The way to understand the behavior of non-deterministic Turing machines is to assume that whenever there is a choice of which transition to take, new Turing machines are started to follow each of the alternate choices. The effect is to run an arbitrary number of machines in parallel. The suprising observation is that even with this additional power, there are no new computable functions.

Theorem: Every nondeterministic TM has an equivalent deterministic TM.
Theorem: A language is decidable iff some nondeterministic Turing machine decides it

NP designates the class of problems that are solved in polynomial time on non-deterministic Turing machines. It is an open question whether or not P = NP. There is however, a great deal of evidence to suggest that P≠NP.

**Table 6:** Running time for programs of various orders

| **Name** | **Running Time Function** | **Order** | **Example:** n=256 (size of input) 1 microsec/instruction $1 \times 10^{-6}$ sec/instruction |
|---|---|---|---|
| Constant time | c | O(1) | |
| Linear time | an + b | O(n) | 0.0025 sec |
| Quadratic time | $a n^2 + b n + c$ | $O(n^2)$ | 0.065 sec |
| Polynomial time | $a n^k + ...$ | $O(n^k)$ | 17 sec (k=3) |
| Exponential | $a k^n + ...$ | $O(k^n)$ | $3.67 \times 10^{61}$ centuries (k=2) |

*Infinite hierarchy of oracles.* The Urim and Thummin on the breastplate on the high priest's ephod provided ancient Israel an oracle that could answer yes or no. Such oracles have been studied in the context of Turing machines. The addition of an oracle to a Turing machine creates a new class of uncomputable or undecidable questions for which we can then assume the existence of an oracle which leads to a new class of uncomputable functions ad infinitum. There is no escape from this infinite hierarchy Turing machines and their undecidable questions.

## What does it mean?

Is it fair/responsible/accurate to describe humans using Turing machines or computable functions? The answer, at least partly, depends on what is meant by describing humans as finite and God as infinite. It would seem reasonable that the finiteness of humans imply finite description and possibly description by a TM. This question is also related to the mind-body problem in philosophy and to the nature of the hunan soul. The mind-body problem is to determine whether the mind is simply a part of the body or if it is something separate from the body. Adventists interpret the creation story and other Biblical texts as asserting that human beings are living souls created from the dust of the ground and animated with the breath of God rather than a soul inserted by God into a body. This approach does not rule out human beings as realizations computable functions. In fact, the notion of a resurrection with a new body but without loss of identity is easily captured by picturing the essence of a human as a function (as a computer program in software) loaded up on new hardware.

My paper on intelligent design, miracles, and oracles applies computational theory to these three areas. The results are negative. Under appropriate assumptions, intelligent design will not in general be able to determine that an artifact is not of intelligent design, miracles are not recognizable by finite humans, and oracles do not provide an escape from uncertainty. But we do not need computational theory to tell us that miracles as events with a supernatural cause are not recognizable [Chavez].

David Hilbert's challenge was to define the notion of an effective method and the Church-Turing thesis is an answer. However, computability is not enough for as Table 6 indicates, programs with exponetial running times will not complete within acceptable time limits. For the problems, we must accept approximate computations with polynomial running times. Many of the key theorem computational theory are negative results, results that reveal limits on what can be done rather than what can be done.

The traditional view of functions as rules, which has been given new importance in computer science where they are called algorithms and are written into computer programs. Computational complexity separates functions into two classes, computable functions and uncomputable functions. Perhaps a third category is necessary, time-varying functions. Functions whose behavior changes over time.

The following are a sample of the discussion and essay questions on this topic:

- The Church-Turing Thesis asserts that all finite processes are describable as Turing machines. Are humans describable by Turing machines? Are intelligence, creativity, spirituality, love, and emotion describable as computable functions? If computer based intelligence equals or outstrips human intelligence, what be the social consequences?
- Since non-deterministic and deterministic Turing machines compute the same class of functions, does the concept of free-will make sense?
- Gödel's incompleteness theorem seems to suggest that we need something more than logic, but the Church-Turing Thesis seems to suggest that we do not have it. Just what is it?
- Are miracles verifiable? Are they supernatural events or ordinary events occurring in n-dimensional Euclidean space? Are the fundamental questions of life decidable? Are the answers recognizable? Is intelligent design decidable/recognizable?
- Some geometric constructions have been proved to be impossible. Gödel's incompleteness theorem shows that some truths are unprovable and the Church-Turing thesis makes some functions uncomputable. Do they suggest that a completely rational Deity is also limited and that the "omni" properties don't make sense? What would a computational approach to theology be like?
- What are the natural roles for religion and reason? Can you find a solution among what Ellen White has written about the relationship between the mind, body, and spirit?

## Summary and Conclusions

Students in American Adventist schools today live in a world that is vastly different from that of just a few years ago. Bronk's statement is equally applicable to the changes that Adventists in North America have seen in the past 50 years.

> "Previously for an European there were only two questions concerning truth of religion: truth of religion in general and truth of Christianity in particular. It was somehow known in advance that other religions are false. Today one faces the multitude of religions and each of them comes with the claim to being true. Thus, a person now needs not only to decide whether it is worthwhile to be religious at all but she has a choice between various competing religions. ... The believers, especially those in the main religious traditions, attribute truth to their religious statements and this truth is for them so important that they choose rather to die than to deny it." - Andrzej Bronk.

Religion once had a monopoly and was indistinguishable from government, culture, and learning. Today, each religion competes as with other religions as well as with government, culture, and learning. While the evangelical and fundamentalist movements would like to restore religion to its former central position in society, the conservative nature of religion places those movements at a disadvantage as they are not able to keep up with the rapid pace of change that is a result of scientific research, cultural changes due to emigration, improved world wide communication, and globalization. The course described in this paper points out some of those areas where the language, logic and the world view of religion must change if it is to compete effectively.

Central to any appliction of the concepts of the paper is the assumption or presupposition of the finiteness of human beings. If the human soul, spirit, consciousness or whatever it is called, transcends the finite or is non-rational, then the limitations of Gödel's incompleteness theorem and the Church-Turing thesis do not apply to human activity. The properties of logic with infinite sentences and processes that accept infinite inputs are areas of study that would have implications for beings with transcendent properties. Just as the Church-Turing thesis cannot be established by a proof, whether or not humans are finite cannot be settled by a proof. If the limits seem unacceptable, then a more powerful and effective definition of computation (or deduction) must be offered.

Even with the limitation of finiteness, humans do have the capacity to conceive of and touch the infinite. Just as not a single irrational number (such as $\pi$, for example) can be computed though its digits may be computed to any degree of accuracy. This is an example of what is labeled "approximatable functions" in Table 5. Mystical events recorded in revealed literature may be how finite humans acquire approximate information regarding the divine.

It may well be that the key contribution of western thought is in its unwillingness to accept paradox, inconsistency, and incompletenes, and its choice to resolve them through the free and open search for truth, knowledge, and understanding (see [Mahbubani] for an Asian perspective).

Why should we study ancient documents and in particular the Bible when little can be done to validate them? For one thing, the documents remind us that little has changed in human behavior and values for thousands of years. For another, that for all our supposed sophistication we still cannot answer fundamental questions and the offered answers have not changed much. Organized religion provides some of the few opportunities to engage in fundamental dialog over the question of how shall we live and religious documents provide some of the most inspiring suggestions. Religion provides a mystical authority to challenge both corrupt authority and to question the social status quo. The following quote may be helpful in this context.

> Our view on this matter is quite simply that neither science nor religion nor pseudoscience offer a product that is satisfactory to all customers. The wares are not just attractive enough. In some cases the beliefs are not useful in the way that people want to use them. For example, many people have a deep-seated psychological need for security and turn to conventional religion for myths of all-powerful and beneficent Beings who will attend to these needs for protection. Science with its mysterious and potentially threatening pronouncements about black holes, the "heat death" of the universe, evolution from lower being, nuclear holocausts, and the like, offers anything but comfort to such primal needs and, as a result, loses customers to the competition. Basically, beliefs thrive because they are useful. And the plain fact is that there is more than one kind of usefulness. -- John L. Casti in *Reality Rules: picturing the world in mathematics* vol 2 *The frontier* Wiley Inter-Science 1992.

As we pass from mathematics though science, the humanities, and arrive at religion, our notions of truth and proof move from precise, objective and effective standards and methods to vague, subjective, and shifting standards and methods. It seems contradictory to require a lower standard of truth for those issues that have eternal consequence. One approach to resolution is to subject both our faith and our learning to the highest possible standards of truth and proof. Historically, it has been faith that failed the test. An alternate approach to the resolution of this apparent contradiction within the Christian paradigm is to require that we apply the same standard to God that He expects of us - Micah 6:8 Act justly, to love mercy, and walk humbly with thy God - in His case, that He act with justice, mercy, and with a deep understanding of His creation. God, the Creator must be a loving, caring, redemptive God.

> *If in our search for truth and understanding we encounter God, then rejoice. If we do not, then the only God worth worshiping will understand and take that into account.*

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